Robust control and observation of nonlinear processes using discontinuities

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Overview

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Introduction

- Discontinuities are useful for dealing with uncertainties and perturbations in control and observation: Sliding Mode control, switching control, hybrid control, ...
- A possible explanation: Discontinuities are simple models of a large class of signals and help in the estimation and compensation of uncertainties and perturbations.
- Objective:
  - Illustrate this in three control/estimation problems.
  - Some lessons learned from simple bioprocesses and how discontinuities can help in their solution.

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- Estimation of states and (unknown) inputs (e.g. reaction rates, faults, ...) is an important topic.
- Challenge: input signals do not have a finite dimensional model $\Rightarrow$ continuous observers can only approximately estimate them, using:
  - High Gains, or
  - Finite dimensional signal models $\Rightarrow$ increases the observer dimension.
- But an Observer with discontinuous output injection term solves exactly the problem for the class of Lipschitz continuous inputs! $\Rightarrow$ Simple Observer.

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Outline

Problem formulation

Nonlinear (second order) system [Moreno and Dochain, 2013]

$$\dot{z}_1 = g_1(z_1, z_2, u)$$
$$\dot{z}_2 = g_2(z_1, z_2, u)$$
$$y = h(z_1, z_2)$$

- state: $z = [z_1, z_2] \in \mathcal{Z} \subseteq \mathbb{R}^2$
- unknown input: $u \in U \subset \mathbb{R}$.
- $\mathcal{Z}$, U compact and connected.
- $g_i(z_1, z_2, u)$ $(i = 1, 2)$, $h(z_1, z_2)$ smooth functions.
- Measured variable: y
- Problem: Using y estimate robustly and in finite time both z and u.

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Model extension and Observability

- Assumption: $u(t)$ Lipschitz continuous, i.e. $|\dot{u}(t)| \leq \alpha$.
- State Extension: $z_3 = u$, $\dot{z}_3 = \dot{u} = g_3(t)$, where $g_3(t)$ is *unknown*, integrable and bounded, i.e. $|g_3(t)| \leq \alpha$,
- Assumption: Strong Observability. i.e. The observability map

$$\mathcal{O}(z) = \begin{bmatrix} h(z) \\ L_g h(z) \\ L_g^2 h(z) \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}$$

  - invertible,
  - independent of $\dot{u}$.

$\Leftrightarrow$ Observability of (z_1, z_2) for any unknown u + "observability" of u.

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Observability form

With transformation $x = \mathcal{O}(z)$

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\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= K(x) + U(x, \dot{u}) \\
y &= x_1 \, ,
\end{aligned}
$$

- $K(x)$ *known* term,
- $U(x, \dot{u})$ *uncertain* term, depending on unknown signal $\dot{u} = g_3(t)$.
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Solvability of the problem: Discussion

- For a continuous observer is impossible to estimate z and u.
- Reason: The class of signals $|\dot{u}(t)| \leq \alpha$ is too large! (infinite dimensional).
- You need a finite dimensional model of u for convergence $\rightarrow$ complex observer!
- But a discontinuous observer can $\rightarrow$ Magic of discontinuity! $\rightarrow$ simple observer
- Discontinuity is a simple model of a large class of signals!

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Observer with discontinuous injection

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- $\lfloor z \rceil^p = |z|^p \operatorname{sign}(z)$
- Estimates x in finite time for all $|\dot{u}(t)| \leq \alpha$.
- Gains $k_1 > 0$, $k_2 > 0$ and $k_3 > 0$, $L > 0$ appropriately selected.
- Critical term: sign function $\lfloor \hat{x}_1 - x_1 \rceil^0$.
- Structure borrowed from Levant's differentiator (A. Levant 2003).
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Observer in original coordinates

$$
\begin{bmatrix} \dot{\hat{z}}_1 \\ \dot{\hat{z}}_2 \\ \dot{\hat{u}} \end{bmatrix} = \begin{bmatrix} g_1(\hat{z}_1, \hat{z}_2, \hat{u}) \\ g_2(\hat{z}_1, \hat{z}_2, \hat{u}) \\ 0 \end{bmatrix} +
$$

$$
- J_{\mathcal{O}}^{-1}(\hat{z}_1, \hat{z}_2, \hat{u}) \begin{bmatrix} Lk_1 \lfloor \hat{y} - y \rceil^{\frac{2}{3}} \\ L^2 k_2 \lfloor \hat{y} - y \rceil^{\frac{1}{3}} \\ L^3 k_3 \lfloor \hat{y} - y \rceil^{0} \end{bmatrix}
$$

$$
\hat{y} = h(\hat{z}_1, \hat{z}_2)
$$

$J_{\mathcal{O}}^{-1}(\hat{z}_1, \hat{z}_2, \hat{u})$ inverse of the Jacobian matrix of observability map $\mathcal{O}(\hat{z}_1, \hat{z}_2, \hat{u})$.

Outline

Unknown input estimation in a bioreactor

$$R:\begin{cases}\dot{X}(t)=\mu(S)X-DX\ , \\ \dot{S}(t)=-\frac{\mu(S)X}{Y}+D\left(S_{in}(t)-S\right)\ , \\ y=X\end{cases}$$

- $X\geq 0$ biomass, $S\geq 0$ substrate concentrations,
- $\mu:\boldsymbol{R}_{+}\to\boldsymbol{R}_{+}$ specific growth rate, given by a Monod law

$$\mu(S)=\frac{\mu_0 S}{S+K_S}=\mu_0 r(S)$$

- $D\geq 0$ dilution rate,
- $S_{in}(t)\geq 0$ unknown input substrate concentration $|\dot{S}_{in}|\leq M$,
- $Y>0$ yield coefficient.

Observer I

$$
\begin{bmatrix} \dot{\hat{X}} \\ \dot{\hat{S}} \\ \dot{\hat{S}}_{in} \end{bmatrix} = \begin{bmatrix} \mu(\hat{S})\hat{X} - D\hat{X} \\ -\frac{\mu(\hat{S})\hat{X}}{Y} + D\left(\hat{S}_{in} - \hat{S}\right) \\ 0 \end{bmatrix} + \\
- J_{\mathcal{O}}^{-1}\left(\hat{X},\, \hat{S},\, \hat{S}_{in}\right) \begin{bmatrix} Lk_1 \left\lfloor \hat{X} - X \right\rceil^{\frac{2}{3}} \\ L^2 k_2 \left\lfloor \hat{X} - X \right\rceil^{\frac{1}{3}} \\ L^3 k_3 \operatorname{sign}\left(\hat{X} - X\right) \end{bmatrix}, \quad (1)
$$

Observer II

where

$$J_{\mathcal{O}}^{-1} = \left[\begin{array}{ccc} 1 & 0 & 0 \\ -\frac{(\mu(S)-D)}{\mu'(S)X} & \frac{1}{\mu'(S)X} & 0 \\ \frac{\eta_2(\mu(S)-D)-\eta_1}{(\mu'(S)X)^2 D}, & \frac{\eta_2(X,S)}{(\mu'(S)X)^2 D}, & \frac{1}{\mu'(S)XD} \end{array} \right]$$

$$\begin{array}{c} \eta_1\left(X,\, S,\, S_{in}\right) = \left(D\left(S_{in} - S\right) - 2\frac{\mu(S)X}{Y} \right) \times \\ \left(\mu'\left(S\right)\right)^2 X + \left(\mu\left(S\right) - D\right)^2 \mu'\left(S\right) X \end{array}$$

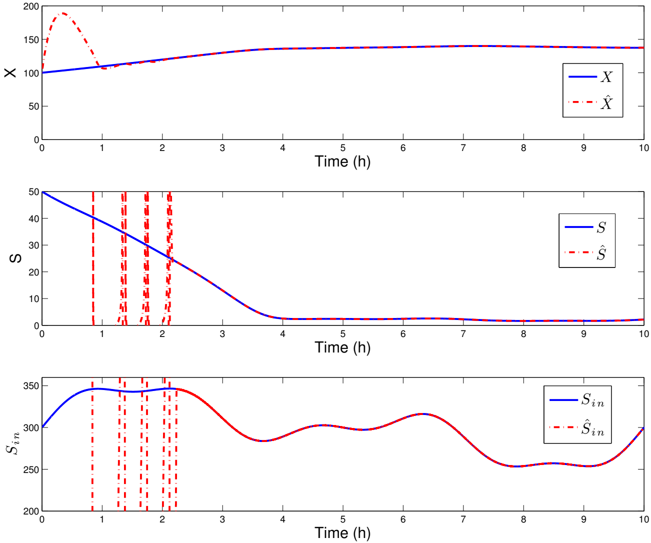
$$\begin{array}{c} \eta_2\left(X,\, S\right) = -\frac{1}{Y}\mu(S)\mu''\left(S\right) X^2 - DS\mu''\left(S\right) X + \\ -\frac{1}{Y}\left(\mu'\left(S\right)\right)^2 X^2 + 2\mu\left(S\right)\mu'\left(S\right) X - 3D\mu'\left(S\right) X\,. \end{array}$$

Outline

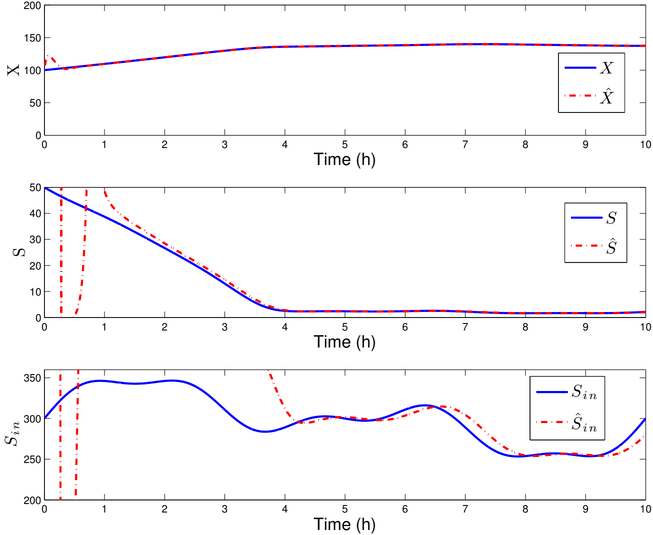
Simulation results

- Parameters: $Y = \frac{1}{2}$, $\mu_0 = \frac{1}{5}$, $K_S = 2$, $D = \frac{1}{2}\mu_0$,
- Initial Conditions: $X_0 = 100$, $S_0 = 50$.
- Observer gains: $k_1 = 13.2$, $k_2 = 50.82$, $k_3 = 13.31$, $L = 2$.
- Unknown input
  $S_{in}(t) = 300 + 30\sin(0.4\pi t) + 30\sin(0.2\pi t) + 10\sin(\pi t)$.
- $S_{in}(t)$ requires a model of dimension 7.

Discontinuous observer



HG observer



200
150
100
50
0
X
0 1 2 3 4 5 6 7 8 9 10
Time (h)
X
$\hat{X}$
50
40
30
20
10
0
S
0 1 2 3 4 5 6 7 8 9 10
Time (h)
S
$\hat{S}$
350
300
250
200
S_{in}
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- Construction of observers is tied to Observability (detectability) properties.
- Local observability is compatible with Global unobservability.
- Observers converge locally but not globally and there is no global observer.
- This phenomenon seems to be common: e.g. chemical reactors, electrical machines (sensorless),...
- A possible solution in case of a finite number of indistinguishable trajectories: reconstruct all possible ones $\Rightarrow$ Discontinuous injection term.
- Initial work: J. Moreno and J. Alvarez, 2013.

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An unobservable bioreactor

$$R : \begin{cases} \dot{X}(t) = \mu(S) X - DX \ , \\ \dot{S}(t) = -\frac{\mu(S) X}{Y} + D(S_{in}(t) - S) \ , \\ y = X \end{cases} \qquad (2)$$

- $\mu : \boldsymbol{R}_{+} \to \boldsymbol{R}_{+}$ non-monotonic, Haldane law

$$\mu(S) = \frac{\mu_0 S}{\frac{S^2}{K_I} + S + K_S} \qquad (3)$$

- At $S^* = \sqrt{K_S K_I}$ achieves its maximum value $\mu^* = \mu(S^*)$.
- $D \geq 0$ dilution rate, $Y > 0$ yield coefficient,
- $S_{in}(t) \geq 0$ unknown input substrate concentration,
- Problem: Using (X, D) estimate (S, S_{in}).

Non monotonic reaction rate

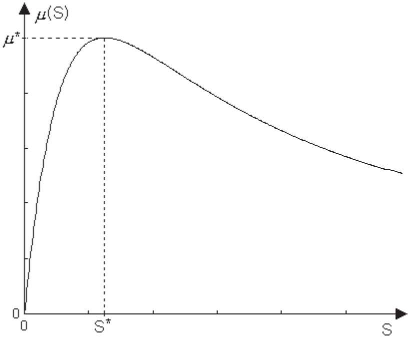

Figure : Haldane Law

Outline

Observability analysis

- Monotonic growth rate $\mu(s)$
  - Observability map globally invertible.
  - System globally Strongly observable + input observable.
  - Global observer for S and S_{in} exists.
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  - For every pair of measured signals $(D(t), X(t))$ in any time interval $t \in [0, T]$, there exist exactly two indistinguishable trajectories!
  - System is not detectable.

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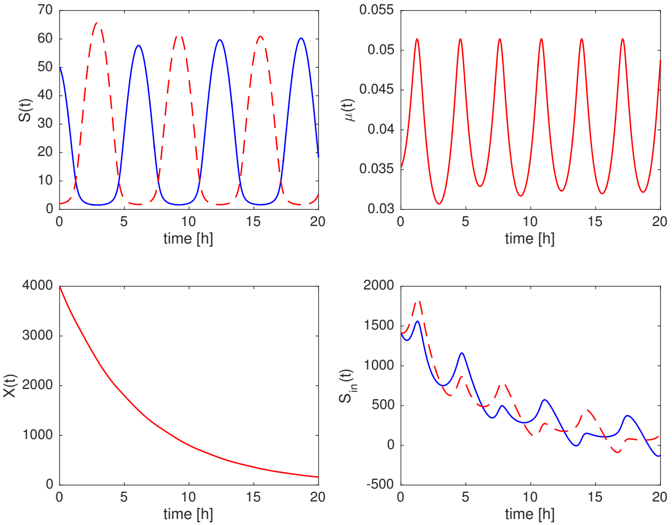
Indistinguishable Trajectories



Outline

Multivalued observers

For (unobservable) systems with a finite number of indistinguishable trajectories:

- A Global observer does not exist.
- Observers may work locally, but not globally.
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- Possible with discontinuous injection terms!

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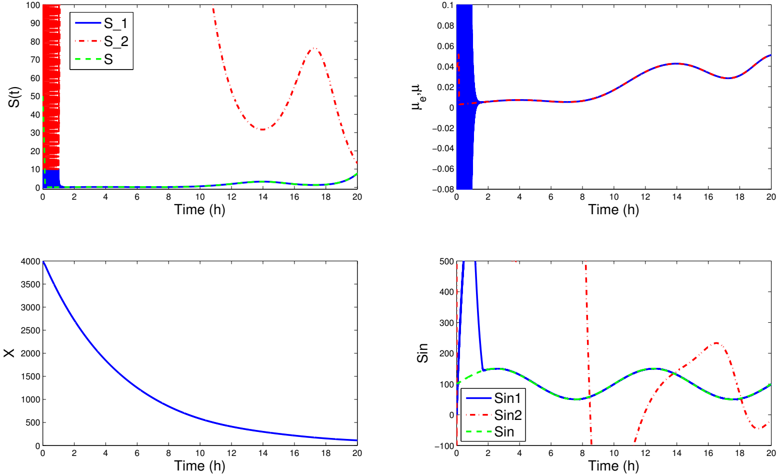
A bivalued observer for the bioreactor

$$
\begin{aligned}
\dot{\hat{X}}(t) &= -k_1\phi_1(e_X) + \hat{\mu}X - DX\,, \quad \hat{X}(t_0) = \hat{X}_0\,,\\
\dot{\hat{\mu}}(t) &= -k_2X\phi_2(e_X)\,, \quad \hat{\mu}(t_0) = \hat{\mu}_0\,,\\
\hat{S}_1(t) &= \frac{K_I(\mu_0 - \hat{\mu}(t)) - \xi}{2\hat{\mu}(t)}\\
\hat{S}_2(t) &= \frac{K_I(\mu_0 - \hat{\mu}(t)) + \xi}{2\hat{\mu}(t)}\,,\\
\xi &= \sqrt{K_I^2(\mu_0 - \hat{\mu}(t))^2 - 4K_SK_I\hat{\mu}^2(t)}\,,\\
e_X &= \hat{X} - X,\\
\phi_1(e_X) &= \gamma_1\lceil e_X\rfloor^{\frac{1}{2}} + \gamma_2 e_X\,, \qquad \gamma_1 > 0\,,\ \gamma_2 \geq 0\,,\\
\phi_2(e_X) &= \frac{\gamma_1^2}{2}\lceil e_X\rfloor^0 + \frac{3}{2}\gamma_1\gamma_2\lceil e_X\rfloor^{\frac{1}{2}} + \gamma_2^2 e_X\,,
\end{aligned}
$$

Bivalued Observer Behavior



Overview

Outline

SISO Regulation and Tracking Problem

SISO smooth, uncertain system

$$\dot{z} = f(t, z) + g(t, z)\, u, \;\; \sigma = h(t, z),$$

- $z \in \mathbb{R}^n$, $u \in \mathbb{R}$, $\sigma \in \mathbb{R}$: sliding variable/tracking error.
- $f(t, z)$ and $g(t, z)$ and n uncertain.
- Control objective: to reach and keep $\sigma \equiv 0$ in finite time.
- Relative Degree ρ w.r.t. σ is well defined, known and constant.
- Reduced (Zero) Dynamics asymptotically stable (by appropriate selection of σ).

The basic DI

Defining $x = (x_1, \ldots, x_\rho)^T = (\sigma, \dot{\sigma}, \ldots, \sigma^{(\rho-1)})^T$, $\sigma^{(i)} = \frac{d^i}{dt^i} h(z,t)$

The regular form

$$\sum_T : \begin{cases} \dot{x}_i = x_{i+1}, & i = 1, \ldots, \rho - 1, \\ \dot{x}_\rho = w(t, z) + b(t, z) u, & x_0 = x(0) , \\ \dot{\zeta} = \phi(\zeta, x) & \zeta_0 = \zeta(0) , \end{cases}$$

$$0 < K_m \leq b(t, z) \leq K_M, \; |w(t, z)| \leq C .$$

Reduced Dynamics Asymptotically stable:

$$\dot{\zeta} = \phi(\zeta, 0) , \quad \zeta_0 = \zeta(0) ,$$

The basic Differential Inclusion (DI)

$$\sum_{DI} : \begin{cases} \dot{x}_i = x_{i+1}, \; i = 1, \ldots, \rho - 1, \\ \dot{x}_\rho \in [-C, C] + [K_m, K_M] u . \end{cases}$$

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Higher Order Sliding Mode (HOSM) Control Solution

Bounded memoryless feedback controller

$$u = \vartheta_\rho(x_1, x_2, \cdots, x_\rho),$$

- A continuous controller ϑ_ρ cannot solve the problem!
- Reason: The class of perturbations/uncertainties is too large.
- ϑ_ρ necessarily discontinuos at $x = 0$ for robustness $[-C, C]$.
- Possible explanation: The discontinuity is a simple model for the class of uncertainties/perturbations.
- Renders $x_1 = x_2 = \cdots = x_\rho = 0$ finite-time stable.
- Motion on the set $x = 0$ is *ρth-order sliding mode.*
- Drawback: Chattering!

Outline

Discontinuous Integral Controller

Homogeneous Discontinuous Integral control

$$\sum_T : \begin{cases} \dot{x}_i = x_{i+1}, & i = 1, ..., \rho - 1, \\ \dot{x}_\rho = u + w(t), & x_0 = x(0) , \end{cases}$$

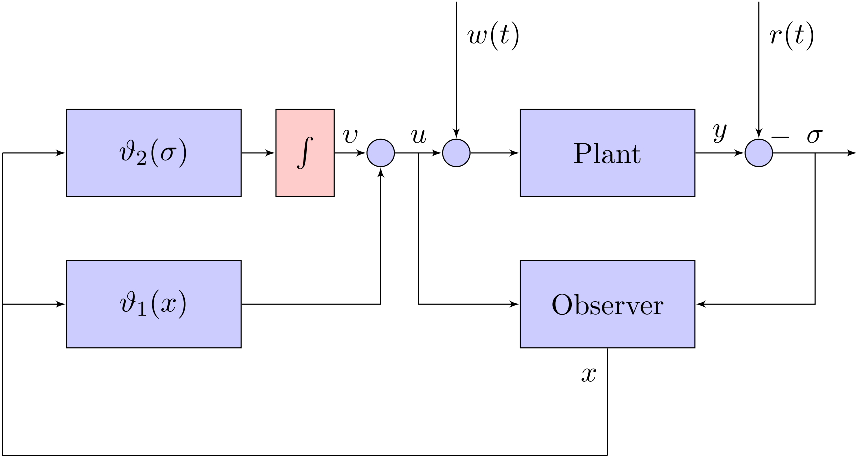
$$|\dot{w}(t, z)| \leq C .$$

$$\begin{aligned} u &= k_1 \vartheta_1(x_1, x_2, \cdots, x_\rho) + x_{\rho+1} , \\ \dot{x}_{\rho+1} &= k_2 \vartheta_2(x_1) , \end{aligned}$$

- $\vartheta_1(\cdot)$ homogeneous,
- $\vartheta_2(\cdot)$ homogeneous of degree 0 (discontinuous!),
- Homogeneity:

$$\vartheta_1\left(\epsilon^{r_1} x_1, \epsilon^{r_2} x_2, \ldots, \epsilon x_{r_\rho}\right) = \epsilon^{\delta} \vartheta_1(x_1, x_2, \ldots, x_\rho) \ \ \forall \epsilon > 0$$

Block Diagram of Discontinuous I-Control



Virtues of Discontinuous Integral Control

- Continuous control signal $u(t)$.
- Continuous (ϑ_2) I-Control rejects/tracks constants $|\dot{w}(t, z)| = 0$.
- Continuous controllers require a model of the references/perturbations to compensate them $\Rightarrow$ Internal Model Principle.
- Discontinuous (ϑ_2) I-Control rejects/tracks Lipschitz perturbations/references! $|\dot{w}(t, z)| \leq C$.
- Discontinuity is a simple model for the class of perturbations/references.
- Requires only x and not $\dot{x}_\rho$.
- $x_{\rho+1}$ estimates perturbation $w \Rightarrow$ for $t \geq T$, $x_{\rho+1}(t) = -w(t)$.
- For $\rho = 1$: Super-Twisting!
- Output feedback: using continuous/discontinuous observer!

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Outline

Magnetic Suspension System

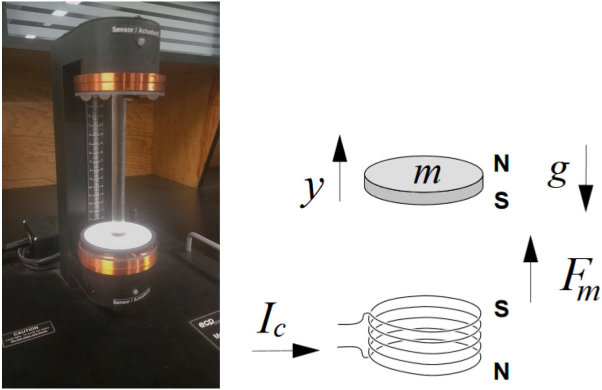


Figure : ECP Model 730: Magnetic Suspension System

Magnetic Suspension System

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{k}{m}x_2 - \frac{aL_0}{2m}\frac{x_3^2}{(a+x_1)^2} + g \\
\dot{x}_3 &= \frac{1}{L(x_1)}\left(-Rx_3 + aL_0\frac{x_2x_3}{(a+x_1)^2} + u\right) \\
L(x_1) &= L_1 + \frac{aL_0}{a+x_1}
\end{aligned}
$$

- $x_1 = y \in \mathbb{R}_+$: position of the disc,
- $x_2 = \dot{y} \in \mathbb{R}$: velocity,
- $x_3 = I_c$: current in the coil,
- $u = V$: voltage.

Discontinuous I-Controller

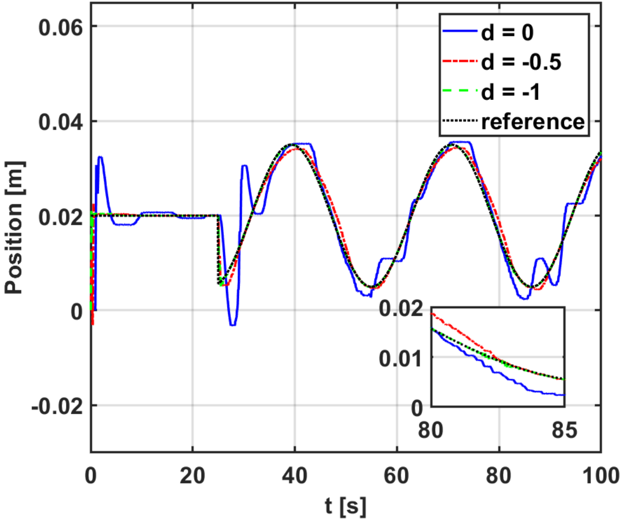
- Control Objective: Position Tracking error $e_1 = y - r(t) \equiv 0$ after finite time.
- Tracking Error Dynamics

$$
\begin{aligned}
\dot{e}_1 =& e_2 \\
\dot{e}_2 =& e_3 \\
\dot{e}_3 =& -k_3 \lambda^{-\frac{d}{4+2d}} \left\lceil \lceil e_3 \rfloor^{\frac{4}{4+2d}} + k_2^{\frac{4}{4+2d}} \lambda^{-\frac{4d}{(4+d)(4+2d)}} \lceil e_2 \rfloor^{\frac{4}{4+d}} + \right. \\
& \left. k_2^{\frac{4}{4+2d}} k_1^{\frac{4}{4+d}} \lambda^{-\frac{12d}{(4+d)(4+2d)}} e_1 \right\rfloor^{\frac{4+3d}{4}} + z + w(t), \\
\dot{z} =& -k_I \lambda \lceil e_1 \rfloor^{\frac{4+4d}{4}} .
\end{aligned}
$$

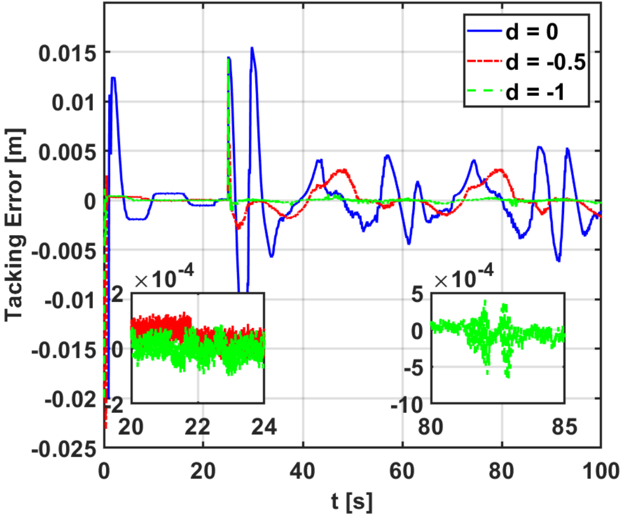
- Homogeneity degree: $d \in [-1, 0]$

- Euler's integration method of fixed-step, sampling time $1 \times 10^{-4}[s]$.
- Gains: $k_3 = 21$, $k_2 = 7$, $k_1 = 3$, $k_I = 2$
- $d = 0$: Lineal controller. $\lambda = 100$.
- $d = -0.5$. Continuous Nonlinear I-Controller. $\lambda = 2$
- $d = -1$: Discontinuous I-controller. $\lambda = 2$

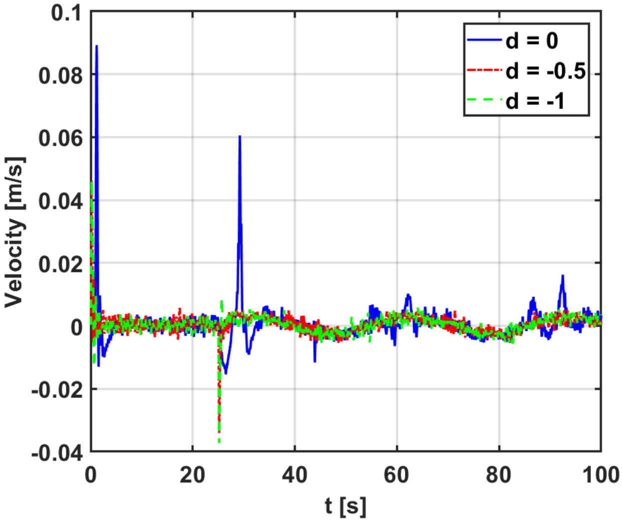
Experiment 1: Position Tracking



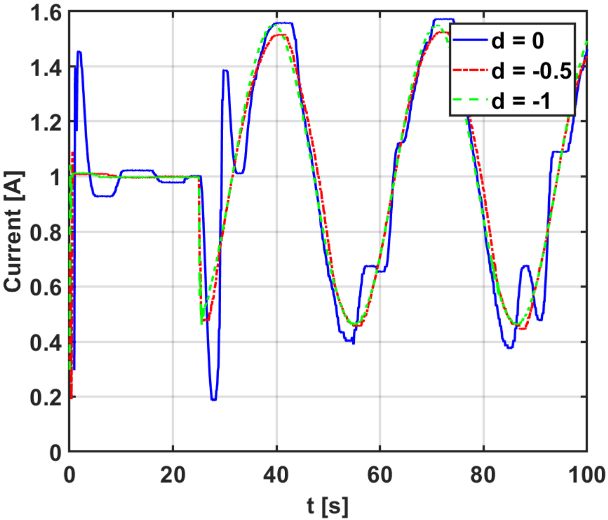
Experiment 1: Tracking error



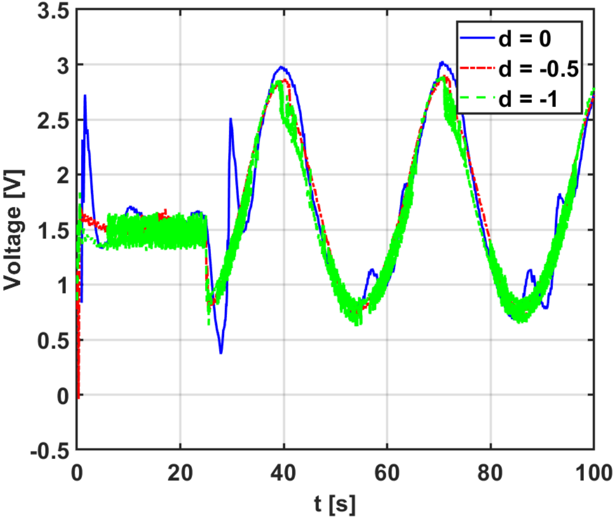
Experiment 1: Velocity



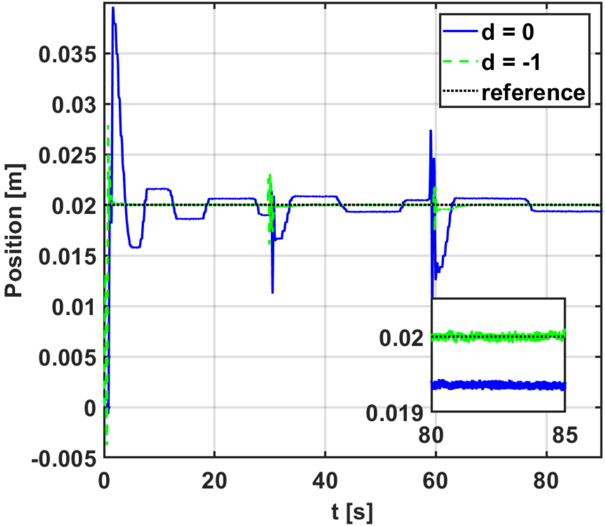
Experiment 1: Current



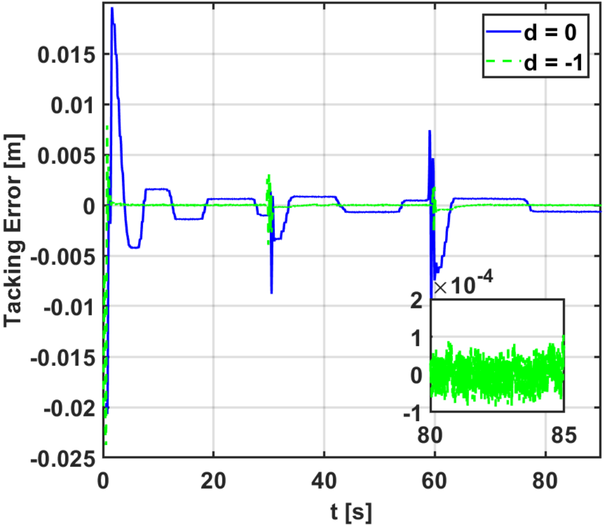
Experiment 1: Control Signal



Exp. 2: Position with varying mass

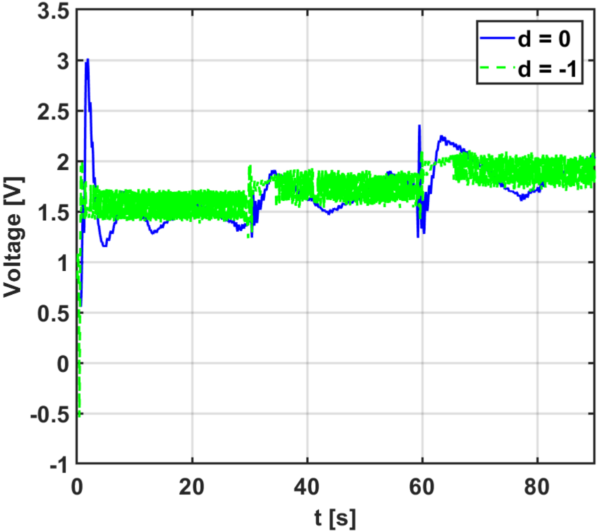


Experiment 2: Regulation error



1

Experiment 2: Control Signal



Overview

Conclusions

- Discontinuities seem to be simple models for a large class of signals.
- They lead to simple solutions for robust control and observation problems.
- Many issues to be studied:
  - Effect of Noise.
  - Implementation: Explicit discretization/Implicit Discretization.
  - Generalizations.
- Extensions and uses of Multivalued Observers.
- We can learn from simple problems and systems!

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